Linear Equation in Two Variables

We are Starting from a Point but want to Make it a Circle of Infinite Radius

Algebraic Representation

The general form of linear equations in two variables x and y is

$$a_1x + b_1y + c_1 = 0$$
and
$$a_2x + b_2y + c_2 = 0$$

where a_1, b_1, c_1, a_2, b_2, c_2 are all real numbers,
This known as the algebraic representation of a system of simultaneous linear equations in two variables.

Graphical Representation of Lingerer Equations

We know that given two lines in a plane, only one of the following three possibilities can happen:
(i)   The two lines intersect at one point.
(ii)   The two lines are parallel
(iii)   The two lines are coincident lines or one line overlaps the other line.

(Intersecting lines)

(Parallel lines)
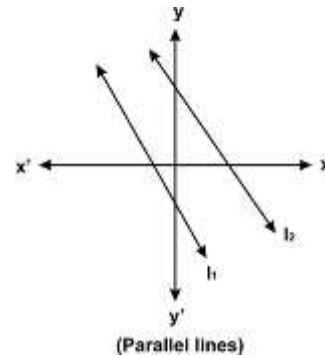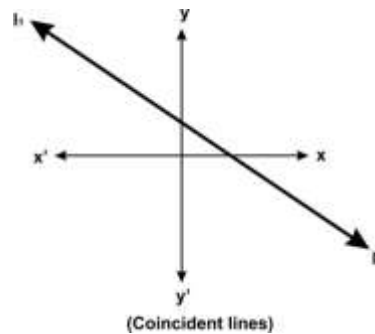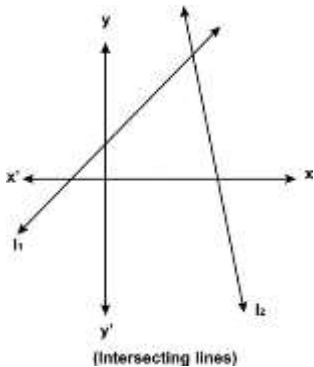
(Coincident lines)

It follows that the pair of linear equations
$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$
will represent

(i)   Intersecting lines, if $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

(ii)   Coincident lines, if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

(iii)   Parallel lines, if $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

Consistent System A system of simultaneous linear equations is said to be consistent, if it has at least one solution.

In-Consistent System A system of simultaneous linear equation is said to be in consistent, if it has no solution.

Methods of solving a pair of linear equations in two variable

Graphical Method

Example 1: find the solution of the equation given below by graphical method

$$2x + y - 6 = 0 \text{ and, } 4x - 2y - 4 = 0$$

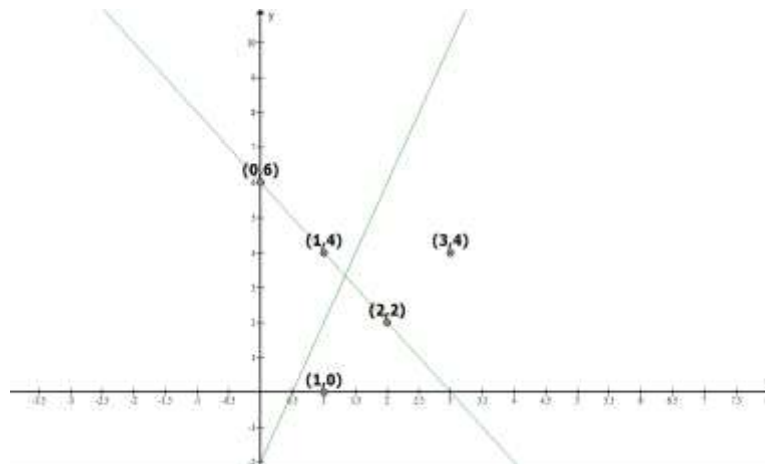Solution: the pair of linear equations is consistent.

$2x + y - 6 = 0$

$y = 6 - 2x$

x	0	1	2
y	6	4	2

And, $4x - 2y - 4 = 0 \Rightarrow y = 4x - 4/2$

x	1	2	3
y	0	2	4

Graphical representation

Example 2: Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.
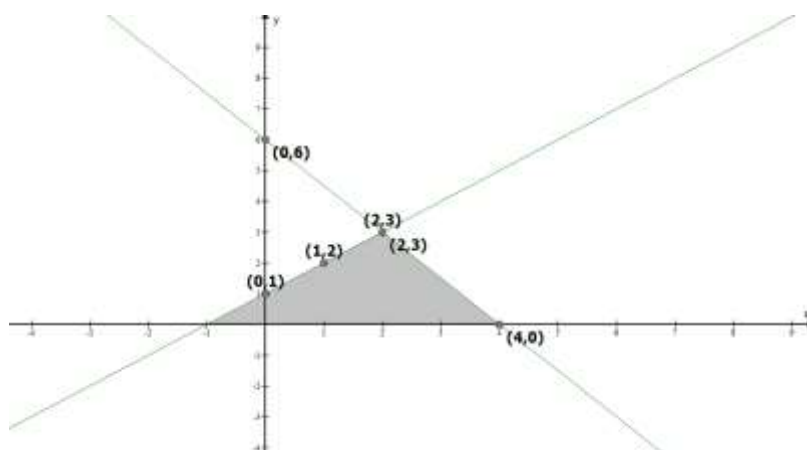
Solution: $x - y + 1 = 0 \Rightarrow x = y - 1$

x	0	1	2
y	1	2	3

$3x + 2y - 12 = 0$

$x = 12 - 2y/3$

x	4	2	0
y	0	3	6

Graphical representation

From the figure, it can be observed that these lines are intersecting each other at point (2, 3) and *x*-axis at (- 1, 0) and (4, 0). Therefore, the vertices of the triangle are (2, 3), (- 1, 0), and (4, 0).

Exercise 3.1

Question based on graphical method

Solve the pair of linear equations by **Graphical Method**

1. Solve the following systems of equations graphically:
    (i)   2x + y − 3 = 0,   (ii)   2x − 3y + 13 = 0
          2x − 3y − 7 = 0   3x − 2y + 12 = 0
    (iii) x − 2y + 11 = 0
          3x − 6y + 33 = 0
2. Show graphically that each one of the following systems of equations has infinitely many solutions:
    $$x − 2y + 11 = 0$$
    $$3x − 6y + 33 = 0$$
3. Show graphically that each one of the following systems of equations is in-consistent (i.e. has no solution):
    $$3x − 4y − 1 = 0$$
    $$2x − \frac{8}{3}y + 5 = 0$$
4. Solve graphically each of the following systems of linear equations. Also find the coordinates of the points where the lines meet axis of y.
    2x − y − 5 = 0, x − y − 3 = 0
5. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are : y = x, y = 2x and y + x = 6
6. Solve the following system of linear equations graphically and shade the region between the two lines and x-axis.
    3x − 2y − 11 = 0, 2x − 3y + 10 = 0
7. Draw the graphs of the following equations:
    $$2x − 3y + 6 = 0$$
    $$2x + 3y − 18 = 0$$
    $$y − 2 = 0$$
    Find the vertices of the triangle so obtained. Also, find the area of the triangle.

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8. Solve the following system of equations graphically:
$$2x - 3y + 6 = 0$$
$$2x + 3y - 18 = 0$$
Also, find the area of the region bounded by these two lines and y-axis.

9. Solve the following system of equations graphically:
Shade the region between the lines and the y-axis.
$$4x - y = 20$$
$$3x + 2y = 14$$
Determine the vertices of the triangle formed by the lines representing the above equation and the y-axis.

10. Draw the graph of the equation $3x - 2y + 6 = 0$. Find whether the points (2, 6) and (1, 2) lie on this graph or not.

Substitution Method :

Example 3: Solve the following pair of linear equations by the substitution method.
$$x + y = 14 \qquad \text{and} \qquad x - y = 4$$
Solution: $x + y = 14$ ... **(i)** and $x - y = 4$ ... **(ii)**
From equation **(i)**, we get $x = 14 - y$ Putting this value in equation **(ii)**, we get
$(14 - y) - y = 4 \Rightarrow 14 - 2y = 4 \Rightarrow 10 = 2y \Rightarrow y = 5$
Putting this in equation **(i)**, we get $x = 9$
∴ $x = 9$ and $y = 5$

Example 4: Solve $2x + 3y = 11$ and $2x - 4y = -24$ by substitution method and hence find the value of '*m*' for which $y = mx + 3$.
Solution: $2x + 3y = 11$ ... **(i)** and $2x - 4y = -24$ … **(ii)**
From (i) we get $2x = 11 - 3y$
Putting this value in equation second we get
$2x - 4y = -24 \Rightarrow 11 - 3y - 4y = -24 \Rightarrow 7y = -24 - 11 \Rightarrow -7y = -35$
$y = -35/-7 \Rightarrow y = 5$
Putting this value in equation **(i)** we get
$2x = 11 - 3 \times 5 \Rightarrow 2x = 11 - 15$
$2x = -4 \Rightarrow x = -2$
Putting the value of x and y
$y = mx + 3$.
$5 = -2m + 3 \Rightarrow 2m = 3 - 5 \Rightarrow m = -2/2 \Rightarrow m = -1$

Exercise 3.2

Question based on substitution method

Solve the following pair of linear Equations by **Substitution Method :**

1. $2x - y - 5 = 0$
   $x - y - 3 = 0$

2. $2x + y - 5 = 0$
   $x + y - 3 = 0$

3. $2x + y - 11 = 0$
   $x - y - 1 = 0$

4. $4x + 5y = 9$
   $8x + 10y = 18$

5. $x - 4y + 14 = 0$
   $3x + 2y - 14 = 0$

6. $2x + 5y = 16$
   $3x + \dfrac{15}{2}y = 24$

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7.	$2x + 3y = 10$	8.	$x + y = 7$
	$4x + 6y = 12$		$2x - 3y = 9$

9.	$x + 4y = 7$	10.	$2x + 3y - 12 = 0$
	$3x + 6y = 10$		$2x + 3y - 6 = 0$

Elimination Method:

Example 5: Solve the following pair of linear equations by the elimination method :

$$3x + 4y = 10 \text{ and } 2x - 2y = 2$$

Solution: $3x + 4y = 10$ and $2x - 2y = 2$

<u>By elimination method</u>

$3x + 4y = 10$ .... **(i)**

$2x - 2y = 2$ ... **(ii)**

Multiplying equation **(ii)** by 2, we get

$4x - 4y = 4$ ... **(iii)**

$3x + 4y = 10$ ... **(i)**

Adding equation **(i)** and **(iii)**, we get

$7x + 0 = 14 => x = 14/7 = 2$

Putting in equation **(i)**, we get

$3x + 4y = 10 => 3(2) + 4y = 10 => 6 + 4y = 10 => 4y = 10 - 6 => 4y = 4 => y = 4/4 = 1$

Hence, answer is $x = 2$, $y = 1$

Exercise 3.3

Solve the pair of linear equation by <u>**Elimination Method**</u> :

1.	$3x - 7y + 10 = 0$	2.	$0.4xx + 0.3y = 1.7$
	$y - 2x - 3 = 0$		$0.7x - 0.2y = 0.8$

3. $\dfrac{x}{2} + y = 0.8$

$\dfrac{7}{x + \dfrac{y}{2}} = 10$

4. $7(y + 3) - 2(x + 2) = 14$

$4(y - 2) + 3(x - 3) = 2$

5. $\dfrac{x}{3} + \dfrac{y}{4} = 11$

$\dfrac{5x}{6} - \dfrac{y}{3} = -7$

6. $4x + \dfrac{6}{y} = 15$

$3x - \dfrac{4}{y} = 7, \ y \neq 0$

7. $x + \dfrac{y}{2} = 4$

$\dfrac{x}{3} + 2y = 5$

8. $x + 2y = \dfrac{x}{2}$

$2x + y = \dfrac{3}{2}$

9. $3x - \dfrac{y + 7}{11} + 2 = 10$

$2y + \dfrac{x + 11}{7} = 10$

10. $x + y = 7$

$2x + 3y = 17$

Cross-Multiplication Method :

Example 6: Solve the two variables linear equation:
8x + 5y = 11
3x – 4y = 10

Solution: On transposition, we get
8x + 5y – 11 = 0
3x – 4y – 10 = 0

Writing the co-efficient in the following way, we get:

By cross-multiplication method:

x/(5) (-10) – (-4) (-11) = y/(-11) (3) – (-10) (8) = 1/(8) (-4) – (3) (5)

or, x/-50 – 44 = y/-33 + 80 = 1/-32 – 15

or, x/-94 = y/47 = 1/-47

or, x/-2 = y/1 = 1/-1 [multiplying by 47]

or, x = -2/-1 = 2 and y = 1/-1 = -1

Therefore, required solution is x = 2, y = -1

Exercise 3.4

Question based on cross multiplication method:
Solve the pair of linear equations by **Cross-Multiplication Method :**

1. 2x – y = 6
   4x – 2y = 6

2. 4x + 3y = 8
   $6x + \dfrac{9}{2}y = 12$

3. 3x + y = 6
   x + y = 4

4. 3x + 2y = 7
   9x + 4y = 9

5. x + 2y = 5
   2x + 3y = 8

6. 0.2x + 0.3y = 1.3
   0.4x + 0.5y = 2.3

7. x + 2y = - 1
   2x + 3y = 8

8. 3x – 5y + 1 = 0
   x – y + 1 = 0

9. 2x – y = 5
   3x + 2y = 11

10. 2x – 3y = 7
    x + y = 1

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Reducing method

Example 7: Solve the following equation by Reducing

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$$

$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Solution:

Let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$

Putting this in (1) and (2) we get

$2p + 3q = 2$ **(3)**

$4p - 9q = -1$ **(4)**

Multiplying (3) by 2 and subtracting it from (4), we get

$4p - 9q + 1 - 2(2p + 3q - 2) = 0$

$\Rightarrow 4p - 9q + 1 - 4p - 6q + 4 = 0$

$\Rightarrow -15q + 5 = 0$

$\Rightarrow q = \frac{-5}{-15} = \frac{1}{3}$

Exercise 3.5

Question based on Reducing method
Solve the pair of linear equations by ***Reducing method***

1. $\dfrac{1}{2x} - \dfrac{1}{3y} = 5$

    $\dfrac{60}{x} + \dfrac{40}{y} = 19, x \neq 0,\ y \neq 0$

2. $\dfrac{6}{x+y} = \dfrac{7}{x-y} + 3$

    $\dfrac{1}{2(x+y)} = \dfrac{1}{3(x-y)}$, where x + y $\neq$ 0 and x − y $\neq$ 0

3. $\dfrac{22}{x+y} + \dfrac{15}{x-y} = 5$

    $\dfrac{55}{x+y} + \dfrac{45}{x-y} = 14$

4. $\dfrac{5}{x+y} - \dfrac{2}{x-y} = -1$

    $\dfrac{15}{x+y} + \dfrac{7}{x-y} = 10$

5. $\dfrac{4}{x} + 5y = 7$

    $\dfrac{3}{x} + 4y = 5$

6. $\dfrac{1}{3x+y} + \dfrac{1}{3x-y} = \dfrac{3}{4}$

    $\dfrac{1}{2(3x+y)} - \dfrac{1}{2(3x-y)} = -\dfrac{1}{8}$

7. $\dfrac{57}{x+y} + \dfrac{6}{x-y} = 5$

    $\dfrac{38}{x+y} + \dfrac{21}{x-y} = 9$

Problem on Properties:

Exanple 8:For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$2x + 3y = 7$

$(a - b)x + (a + b)y = 3a + b - 2$

Solution: $2x + 3y - 7 = 0$

$(a - b)x + (a + b)y - (3a + b - 2) = 0$

$a_1/a_2 = 2/a-b$

$b_1/b_2 = 3/a+b$ and

$c_1/c_2 = 7/(3a+b-2)$

For infinitely many solutions,$a_1/a_2 = b_1/b_2 = c_1/c_2$

$2/a-b = 7/3a+b-2 => 6a + 2b - 4 = 7a - 7b$

$a - 9b = -4$ ... **(i)**

$2/a-b = 3/a+b$

$2a + 2b = 3a - 3b$

$a - 5b = 0$ ... **(ii)**

Subtracting equation **(i)** from **(ii)**, we get

$4b = 4$

$b = 1$

Putting this value in equation **(ii)**, we get

$a - 5 \times 1 = 0$

$a = 5$

Hence, $a = 5$ and $b = 1$ are the values for which the given equations give infinitely many solutions.

Example 9:For which value of k will the following pair of linear equations have no solution?

$3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

Solution: $3x + y - 1 = 0$

$(2k - 1)x + (k - 1)y - (2k + 1) = 0$

$a_1/a_2 = 3/2k-1$

$b_1/b_2 = 1/k-1$ and

$c_1/c_2 = -1/-2k-1 = 1/2k+1$

For no solutions,

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

$3/2k-1 = 1/k-1 \neq 1/2k+1 => 3/2k-1=1/k-1 => 3k -3=2k -1 => k =2$

Hence, for $k = 2$, the given equation has no solution.

Question based on condition:
1. Find the value of k for which each of the following systems of equations have infinitely many solution.

    1. $kx - 2y + 6 = 0$
        $4x - 3y + 9 = 0$

    2. $2x - 3y = 7$
        $(k + 2)x - (2k + 1)y = 3(2k - 1)$

    3. $kx + 3y = 2k + 1$
        $2(k + 1)x + 9y = 7k + 1$

    4. $2x + (k - 2)y = k$
        $6x + (2k - 1)y = 2k + 5$

    5. $2x + 3y = k$
        $(k - 1)x + (k + 2)y = 3k$

2. Find the value of k for which the following system of equations has no solution.
    1. $x + 2y = 0$
        $2x + ky = 5$

    2. $2x - ky + 3 = 0$
        $3x + 2y - 1 = 0$

3. For what value of k the following system of equations will be inconsistent?
    $$4x + 6y = 11$$
    $$2x + ky = 7$$

4. For what value of α, the system of equations
    $$\alpha x + 3y = \alpha - 3$$
    $$12x + \alpha y = \alpha$$
    Will have no solution?

5. Find the value of k for which the system
    $$kx + 2y = 5$$
    $$3x + y = 1$$
    has (i) a unique solution, and (ii) no solution.

6. For what value of k, the following system of equations will represent the coincident lines?
    $$x + 2y + 7 = 0$$
    $$2x + ky + 14 = 0$$

7. Determine the values of a and b so that the following system of linear equations have infinitely many solutions:
    $$(2a - 1)x + 3y - 5 = 0$$
    $$3x + (b - 1)y - 2 = 0$$

8. Find the value of a and b for which the following system of linear equations has infinite number of solutions.
    $$2x - 3y = 7$$
    $$(a + b)x - (a + b - 3)y = 4a + b$$

9. Find the values of p and q for which the following system of linear equations has infinite number of solutions.
    $$2x + 3y = 7$$
    $$(p + q)x - (2p + q)y = 3(p + q + 1)$$

Words Problems

__Question based on to find the cost of the object__

Example 10: The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, she buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

Solution:

Let cost of each bat = Rs x

Cost of each ball = Rs y

Given that coach of a cricket team buys 7 bats and 6 balls for Rs 3800.

$7x + 6y = 3800$

$6y = 3800 - 7x$

Dividing by 6, we get

$y = (3800 - 7x)/6$ … **(i)**

Given that she buys 3 bats and 5 balls for Rs 1750 later.

$3x + 5y = 1750$

Putting the value of y

$3x + 5((3800 - 7x)/6) = 1750$

$18x + 19000 - 35x = 10500 => -17x = 10500 - 19000 => -17x = -8500 => x = -8500/-17$

$x = 500$

Putting this value in equation **(i)** we get

$y = (3800 - 7 \times 500)/6 => y = 300/6$

$y = 50$

Hence cost of each bat = Rs 500 and cost of each balls = Rs 50.

Exercise 3.7

1. A man sold a chair and a table together for Rs. 1520 there by making a profit of 25% on chair and 10% on table. By selling them together for Rs. 1535 he would have made a profit of 10% on chair and 25 % on table. Find the cost price of each.

    Three bags and 4 pens together cost Rs. 257 while 4 bags and 3 pens together cost Rs. 324. Find the total cost of 1 bag and 10 pens.

2. Two audio cassettes and three video cassettes cost Rs. 340. But three audio cassettes and two video cassettes cost Rs. 260. Find the price of an audio cassette and that of a video cassette.

3. Three chairs and two tables cost Rs. 1850. Five chairs and three tables Rs. 2850. Find the cost of two chairs and two tables.

4. If we buy 2 tickets from station A station B, and 3 from station A to station C, we have to pay Rs. 795. But 3 tickets from station A to B and 5 tickets from A to C cost a total of Rs. 1300. What is the fare from station A to B and that from station A to C?

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5. 4 chairs and 3 tables cost Rs. 2100 and 5 chairs and 2 tables cost Rs. 1750. Find the cost of a chair and a table separately.

6. 2 tables and 3 chairs together cost Rs. 2000 whereas 3 tables and 2 chairs together cost Rs. 2500. Find the total cost of 1 table and 5 chairs.

7. If 37 pens and 53 pencils together cost Rs. 320, while 53 pens and 37 pencils together cost Rs. 400. Find the cost of a pen and that of a pencil.

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8.   On selling a tea set at 5% loss and a lemon set at 15% gain, a crockery seller gains Rs. 7. If he sells tea set at 5 % gain and lemon set at 10% gain , he gains Rs. 13. Find the actual price of the tea set and the lemon set.

9.   Two audio cassettes and 3 video cassettes cost Rs. 830 and 5 audio cassettes and 2 video cassettes cost Rs. 700 respectively. Determine the cost of 4 audio cassettes and 5 video cassettes.

10. 7 audio cassettes and 3 video cassettes cost Rs. 1110, while 5 audio cassettes and 4 video cassettes cost Rs. 1350. Find the cost of an audio cassettes and that of a video cassettes.

11. The cost of 7 pens and 4 pencils is Rs. 23 and cost of 6 pens and 2 pencils is Rs. 19. Find the cost of one pen and one pencil separately.

12. A man sold a chair and a table for Rs. 760 thereby making a profit of 25% on the chair and 10% on the table. By selling them for Rs. 767.50, he would have realized a profit of 10% on chair and 25% on table. Find the cost price of chair and table separately.

13. From Delhi station if we buy 2 tickets to station A and 3 tickets to station B, the total cost is Rs. 77, but if we buy 3 tickets to station A and 5 tickets to station B, the total cost is Rs. 124. What are the fares from Delhi to station A and to station B?

Question based on to find the numbers:

Example 11: The difference between two numbers is 26 and one number is three times the other. Find them.

Solution: Let larger number $= x$

Smaller number $= y$

The difference between two numbers is 26

$x - y = 26$

$x = 26 + y$

Given that one number is three times the other

So $x = 3y$

Putting the value of x we get

$26y = 3y \Rightarrow -2y = -26 \Rightarrow y = 13$

So value of $x = 3y$

$x = 3 \times 13 = 39$

Hence the numbers are 13 and 39.

Exercise 3.8

1.   Two numbers differ by 2 and their product is 360. Find the numbers.

2.   Two numbers are in the ratio of 3 : 4. If 8 is added to each number, they become in the ratio of 4 : 5. Find the numbers.

3.   The sum of two numbers is 15 and sum of their reciprocals is $\dfrac{3}{10}$. Find the numbers.

4.   The sum of two numbers is 16 and sum of their reciprocals is $\dfrac{1}{3}$. Find the numbers.

5.   Two numbers are in the ratio of 5 : 6. If 8 is subtracted from each of the number, they become in the ratio of 4 : 5. Find the numbers.

6.   Find the whole number which when decreased by 20 is equal to 69 times the reciprocal of the number.

Question based angles:

Example 12: The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

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Solution: Let first angle = x

And second number = y

As both angles are supplementary so that sum will 180

$x + y = 180$

x = 180 - y ... **(i)**

Difference is 18 degree so that

$x - y = 18$

Putting the value of x we get

$180 - y - y = 18$

$- 2y = -162$

$y = -162/-2$

$y = 81$

Putting the value back in equation **(i)**, we get

$x = 180 - 81 = 99$Hence, the angles are 99° and 81°.

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Exercise 3.9

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Question based angles:

1. In $\triangle ABC$, $\angle A = x°$, $\angle B = 3x°$, $\angle C = y°$. If $3y - 5x = 30$, prove that the $\triangle$is right angled.

    [Ans. 90°]

2. Find the four angles of a cyclic quadrilateral ABCD in which $\angle A = (2x - 1)°$, $\angle B = (y + 5)°$, $\angle C = (2y + 15)°$, $\angle D = (4x - 7)°$.

    [Ans. $\angle A = 65°$, $\angle B = 55°$, $\angle C = 115°$, $\angle D = 125°$]

3. The largest angle of a triangle is twice the sum of the other two. The smallest is one-fourth of the largest. Determine all the angles in degrees.

    [Ans. 120°, 30°, 30°]

4. The largest angle if a $\triangle$ is equal to the sum of the other two angles. The smallest angle is 1 / 4 of the largest one. Find the angles of the $\triangle$.

    [Ans. 90°, 67 ½ °, 22 ½ °]

5. In a triangle the sum of two angles is equal to the third angle. If the difference between two angles is 30°, find the angles of the triangle.

    [Ans. 60°, 30°, 90°]

Question based on to find the age of the object

Example 13 : Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Solution: Let present age of Jacob = x year

And present Age of his son is = y year

Five years hence,

Age of Jacob will = $x + 5$ year

Age of his son will = $y + 5$year

Given that the age of Jacob will be three times that of his son

$x + 5 = 3(y + 5)$

Adding 5 both side, we get

$x = 3y + 15 - 5$

$x = 3y + 10$ … **(i)**

Five years ago,

Age of Jacob will = $x - 5$ year

Age of his son will = $y - 5$ year

Jacob's age was seven times that of his son

$x - 5 = 7(y - 5)$

Putting the value of x from equation **(i)** we get

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NTSE, NSO
Diploma, XI Entrance

[39]

CLASS - X
Mathematics

Linear Equation in
Two Variables

$3y + 10 - 5 = 7y - 35$
$3y + 5 = 7y - 35$
$3y - 7y = -35 - 5$
-4y = - 40
$y = - 40/- 4$
$y = 10$ year
Putting the value of y in equation first we get
$x = 3 \times 10 + 10$
$x = 40$ years
Hence, Present age of Jacob = 40 years and present age of his son = 10 years.

Exercise 3.10

1. The ages of two friends A and B differ by 3 years. A's father D is twice as old as A. B is twice as old as his sister C. The ages of C and D differ by 30 years. Find the ages of A and B.

2. If twice the son's age in years is added to the father's age, the sum is 70. But if twice the father's age is added to the son's age, the sum is 95. Find the ages of father and son.

3. I am three times as old as my son. Five years later, I shall be two and a half times as old as my son. How did am I and how old is my son?

4. Ten years ago, father was 12 times as old as his son and ten years hence, he will be twice as old as his son will be. Find their present ages.

5. Four years ago a father was six times as old as his son. Eight years hence, the father will be thrice as old as his son. Find their present ages.

6. Five years ago I was thrice as old as my son and ten years later I shall be twice as old as my son. How old are we now?

7. The age of father is 3 years more than three times the age of the son. Three years hence, father's age will be 10 years more than twice the age of the son. Determine their present ages.

8. The ratio of the present ages of Anu and Ajay is 7 : 9. Nine years ago, the ratio of their ages was 2 : 3. Find their present ages.

9. The ages of A, B and C together totals 185 years. B is twice as old as A and C is 17 years older than A. How old are they?

10. The present age of a father is equal to the sum of the ages of his 5 children. 12 years hence, the sum of the ages of his children will be twice the ages of their father. Find the present age of the father.

11. A man says to his son "seven year ago I was seven times as old as you were, and three years hence I shall be three times as old as you". Find their ages.

12. A father's age is equal to the sum of the ages of his 5 children. In 15 years his age will be only one half of the sum of their ages. How old is the father?

13. Father is six times as old as his son. Four years hence he will be four times as old as his son. Find their present ages.

14. Six years hence a man's age will be three times his son's age and three years ago he was nine times as old as his son. Find their present ages.

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15. A year ago, the father was 8 times as old as his son. Now his age is square of his son's age. Find their present ages.

16. The age of father is four times the age of son. Five years hence, the age of father will be three times the age of son. Find their present ages.

17. One year ago a man was four times as old as his son. After 6 years his age exceeds twice his son's age by 9 years. Find their present ages.

18. Ten years ago, the age of a man was four times that of his son. Five years ago, the age of the man was three times that of his son. Find their present ages.

Question based on Two digit Number:

Example 14: The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

Solution: Let the unit digit and tens digits of the number be x and y respectively.

Then, number $= 10y + x$

Number after reversing the digits $= 10x + y$

According to the question,

$x + y = 9$ ... **(i)**

$9(10y + x) = 2(10x + y) => 88y - 11x = 0 => -x + 8y = 0$ ... **(ii)**

Adding equation **(i)** and **(ii)**, we get

$9y = 9 => y = 1$

Putting the value in equation **(i)**, we get , $x = 8$

Hence, the number is $10y + x = 10 \times 1 + 8 = 18$.

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Exercise 3.11

</div>

1. A number consists of two digits whose sum is 9. If 27 is added to the number, digits change their places. Find the number.

2. In a two digit number, the unit's digit is twice the ten's digit. If 27 is added to the number, the digits interchange their places. Find the number.

3. In a two digit number, the ten's digit is three times the unit's digit. When the number is decreased by 54, the digits are reversed. Find the number.

4. The sum of digits of a two digit number is 8 and the difference between the number and that formed by reversing the digits is 18. Find the number.

5. The sum of a two digit number and the number obtained by reversing the order of its digits is 121, and the two digits differ by 3. Find the number.

6. A two digit number is obtained by either multiplying sum of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2. Find the number.

7. The sum of a two digit number and the number obtained by reversing the order of its digits is 154. If the two digits of the given number differ by 4, find the number.

8. A number of two digits becomes 5 / 4 of itself if its digits are interchanged. If the difference of the digits is 1, find the number.

9. There is a number to the sum of whose digits you add 7 the result will be three times the left hand digit. If from the number itself you subtract 18, the digits will be reversed. Find the number.

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Question based on to find the friction:

Example 15: If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes 1/2 if we only add 1 to the denominator. What is the fraction?

Solution : Let the fraction be x/y
According to the question,

$x + 1/y - 1 = 1 \Rightarrow x - y = -2$ ... **(i)**

$x/y + 1 = \frac{1}{2} \Rightarrow 2x - y = 1$ ... **(ii)**
Subtracting equation **(i)** from equation **(ii)**, we get
$x = 3$ ... **(iii)**
Putting this value in equation **(i)**, we get
$3 - y = -2$
$-y = -5$
$y = 5$
Hence, the fraction is 3/5

Exercise 3.12

1. If 2 is added to the numerator of a fraction, it reduces to 1 / 2 and if 1 is subtracted from the denominator, it reduces to 1 / 3. Find the fraction.

2. A fraction becomes 4 / 5 if 1 is added to each of the numerator and the denominator. However, if we subtract 5 from each, the fraction becomes 1 / 2. Find the fraction.

3. If we add 1 in the numerator of a fraction and subtract 1 from its denominator, the fraction becomes 1. If it is also given that the fraction becomes 1 / 2 when we add 1 to its denominator, then what is the fraction?

4. If we add 1 to the numerator and denominator of a fraction we get 2 / 3. If we subtract 1 from numerator and denominator and denominator we get 1 / 2. Find the fraction.

5. In a given fraction, if numerator is multiplied by 2 and denominator is reduced by 5, we get 6 / 5. But, if numerator of the given fraction is increased by 8 and denominator is doubled, we get 2 / 5. Find the fraction.

6. If 1 is added to the numerator and subtracted from denominator of a fraction, it becomes 1. If 1 is added to denominator and subtracted from numerator, it becomes 1 / 2. Find the fraction.

7. A fraction becomes 1 / 3 when 2 is subtracted from numerator but it becomes 1 / 2 when 1 is added to denominator. Find the fraction.

8. Find a fraction becomes 1 / 2 when 1 is subtracted from numerator and 2 is added to denominator, and becomes 1 / 3 when 7 is subtracted from numerator and 2 from denominator.

9. If I add 1 to each of the two given number, then their ratio is 1 : 2. If I subtract 5 from each, the ratio is 5 : 11. Find the numbers.

Question based on fixed charge and charge per unit

Example 16: A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Solution: Let the fixed charge for first three days and each day charge thereafter be Rs x and Rs y respectively.

According to the question,

$x + 4y = 27$ ... (i)

$x + 2y = 21$ ... (ii)

Subtracting equation (ii) from equation (i), we get

$2y = 6$

$y = 3$ ... (iii)

Putting in equation (i), we get

$x + 12 = 27$

$x = 15$

Hence, fixed charge = Rs 15 and Charge per day = Rs 3.

Exercise 3.13

1. The car hire charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 12 km, the charge paid is Rs. 89 and for a journey of 20 km, the charge paid is Rs. 145. What will a person have to pay for traveling a distance of 30 km?

2. The taxi charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 10 km, the charge paid is Rs. 75 and for a journey of 15 km, the charge paid is Rs. 110. What will a person have to pay for traveling a distance of 25 km?

3. Taxi charges consist of fixed charges per day and the remaining depending upon the distance traveled in kilometers. If a person travels 110. km, he pays Rs. 690 and for traveling 200 km, he pays Rs. 1,050. Express the above statements in the form of simultaneous equations and hence, find the fixed charges and the rate per km.

4. Scooter charges consist of fixed charges and the remaining depending upon the distance traveled in kilometers. If a person travels 12 km, he pays Rs. 45 and for traveling 20 km, he pays Rs. 73. Express the above statements in the form of simultaneous equations and hence, find the fixed charges and the rate per km

5. A part of monthly hostel charges in a college is fixed and the remaining depend on the number of days one has taken food in the mess. When a student A takes food for 20 days, he has to pay Rs. 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs. 1180 as hostel charges. Find the fixed charge and the cost of food per day.

6. The car rental charges in a city comprise a fixed charge together with the charge for the distance covered. For a journey of 13 km, the charge paid is Rs. 96 and for a journey of 18 km, the charge paid is Rs. 131. What will a person have to pay for traveling a distance of 25 km.

7. The total expenditure per month of a household consist of a fixed rent of the house and the mess charges, depending upon the number of people sharing the house. The total monthly expenditure is Rs. 3, 900 for 2 people and Rs. 7, 500 for 5 people. Find the rent of the house and the mess charges per head per month.

Question based on speed, distance and time

Example 17: Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

Solution: Let the speed of 1st car and 2nd car be u km/h and v km/h.

Respective speed of both cars while they are travelling in same direction = $(u - v)$ km/h

Respective speed of both cars while they are travelling in opposite directions i.e., travelling towards each other = $(u + v)$ km/h

According to the question,

$5(u - v) = 100 \Rightarrow u - v = 20$ ... **(i)**

$1(u + v) = 100$ ... **(ii)**

Adding both the equations, we get

$2u = 120$

$u = 60$ km/h ... **(iii)**

Putting this value in equation **(ii)**, we obtain

$v = 40$ km/h

Hence, speed of one car = 60 km/h and speed of other car = 40 km/h

Exercise 3.14

1. In a flight of 6000 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 400 km/hour and time increased by 30 minutes. Find the original duration of flight.

2. A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km per hour more, it would have taken half an hour les for the journey. Find the original speed of the train.

3. An express train makes a run of 240 km at a certain speed. Another train whose speed is 12 km/hr less takes an hour longer to cover the same distance. Find the speed of the express train in km/hr.

4. A man travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But, if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.

5. Places A and B are 80 km apart from each other on a highway. One car starts from A and another from B at the same time. If they move in the same direction, they meet in 8 hours and if they move in opposite direction, they meet in 1 hour and 20 minutes. Find the speeds of the cars.

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Question based on Area and work

Example 18: The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Solution : Let length and breadth of rectangle be x unit and y unit respectively.
Area = xy
According to the question,
$(x - 5)(y + 3) = xy - 9$
$\Rightarrow 3x - 5y - 6 = 0$ ... **(i)**
$(x + 3)(y + 2) = xy + 67$
$\Rightarrow 2x - 3y - 61 = 0$ ... **(ii)**
By cross multiplication, we get
$x/305-(-18) = y/-12-(-183) = 1/9-(-10)$
$x/323 = y/171 = 1/19$
$x = 17, y = 9$
Hence, the length of the rectangle = 17 units and breadth of the rectangle = 9 units.

Example 19: 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

Solution: Let the number of days taken by a woman and a man be x and y respectively.
Therefore, work done by a woman in 1 day = $1/x$
According to the question,
$4(2/x + 5/y) = 1$
$2/x + 5/y = 1/4$
$3(3/x + 6/y) = 1$
$3/x + 6/y = 1/3$
Putting $1/x = p$ and $1/y = q$ in these equations, we get
$2p + 5q = 1/4$
By cross multiplication, we get
$p/-20-(-18) = q/-9-(-18) = 1/144-180$
$p/-2 = q/-1 = 1/-36$
$p/-2 = -1/36$ and $q/-1 = 1/-36$
$p = 1/18$ and q = 1/36
$p = 1/x = 1/18$ and $q = 1/y = 1/36$
$x = 18$ and $y = 36$
Hence, number of days taken by a woman = 18 and number of days taken by a man = 36

1. The area of a rectangle gets reduced by 80 sq. units if its length is reduced by 5 units and the breadth is increased by 2 units. If we increase the length by 10 units and decrease the breadth by 5 units, the area is increased by 50 sq. units. Find the length and breadth of the rectangle.

2. 10 men and 15 women together can complete a work in 6 days. It takes 100 days for one man alone to complete the same work.How many days will be required for one woman alone to complete the same work ?

3. If 6 men and 8 boys can do a piece of work in 10 days while 26 men and 48 boys can do the same in 2 days, the time taken by 15 men and 20 boys in doing the same type of work will be :

4. If 3 men or 4 women can do a work in 43 days, how long will 7 men and 5 women take to complete the work ?

Question based on upstream and downstream

Example 19: Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.
Solution: Let the speed of Ritu in still water and the speed of stream be x km/h and y km/h respectively.
Speed of Ritu while rowing
Upstream = $(x - y)$ km/h
Downstream = $(x + y)$ km/h
According to question,
$2(x + y) = 20 \Rightarrow x + y = 10$ ... **(i)**
$2(x - y) = 4 \Rightarrow x - y = 2$ ... **(ii)**
Adding equation **(i)** and **(ii)**, we get
Putting this equation in **(i)**, we get
$y = 4$
Hence, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

Exercise 3.16

1. A boat goes 12 km upstream and 40 km downstream in 8 hours. It can go 16 km upstream and 32 km downstream in the same time. Find the sped of the boat in still water and the speed of the stream.

2. The speed of a boat in still water is 8 km/hour. It can go 15 km upstream and 22 km downstream in 5 hours. Find the speed of the stream.

3. The speed of a boat in still water is 15 km/hr. it can go 30 km upstream and return down stream to the original point in 4 hrs. and 30 minutes. Find the speed of the stream.

4. A sailor goes 8 km downstream in 40 minutes and returns in 1 hr. Determine the speed of the sailor in still water and the speed of the current.

5. A boatman rows his boat 20 km upstream and 16 km downstream in 7 hours. And he can row 32 km upstream and 8 km downstream in 9 hours. Find the speed of the stream and that of the boat in still water.   Ans 6km/h and 2km/h

ANSWERS

Exercise − 3.2

    1. $x = 2$, y = −1   2. $x = 2$, y = 1   3. $x = 4$, y = 3

    4. Infinite many solution x = $\dfrac{9-5y}{4}$ 5. $x = 2$, y = 4

    6. Infinite many solution x = $\dfrac{16-5y}{2}$ 7. NO solution

    8. $x = 6$, y = 1   9. $x = \dfrac{-1}{3}, \ y = \dfrac{11}{6}$ 10. No solution

Exercise − 3.3

    1. $x = -1$, y =1   2. $x = 2$, y = 3   3. $x = 2/5$, y = 3/5

    4. $x = 5$, y = 1   5. $x = 6$, y = 36   6. $x = 3$, y = 2

    7. $x = 3$, y = 2   8. $x = 6/7$, y = −3/14   9. $x = 3$, y = 4

    10. $x = 4$, y = 3

Exercise − 3.4

    1. No solution   2. Infinite many solution x = 8−3y/4

    3. $x = 1$, y =3   4. $x = -5/3$, y =6   5. $x = 1$, y =2

    6. $x = 2$, y = 3   7. $x = 35$, y = −18   8. $x = -2$, y = −1

    9. $x = 3$, y = −1   10. $x = 2$, y = −1

Exercise − 3.5

    1. $x = 40/407$, y = 80/21   2. $x = -5/4$, y = −1/4

    3. $x = 0$, y = 3   4. $x = 3$, y = 2   5. $x = 1/3$, y = −1

    6. $x = 1$, y = 1   7. $x = 11$, y = 8

Exercise − 3.6

1.   (1). K = 8/3   (2). K = 4   (3). K = 2

       (4). K = 5   (5). K = 7

2.   (1). K = 4   (2). K = −4/3, k $\ne$ 6

3.   K = 3, k $\ne$ 42/11   4. $\alpha = -6, \propto \ne 6$

5.   (i). K $\ne$ 6,   (ii) k = 6

6.   k = 4   7. a = 17/4, b = 11/5

8.   a = 1, b = 1/5   9. P = −15, q = 21

Exercise − 3.7

1.   Rs. 600 and Rs. 700   2.   Rs. 155

3.   Audio cassette Rs. 20 and Video cassette Rs 100

4.   Rs. 1700   5.   A to B Rs. 75 and A to C Rs. 215

6.   Rs. 150, Rs. 500   7.   Rs. 1700]

8.   Rs. 6.50, Rs. 1.50   9.   Rs. 100, Rs. 80

10.   Rs. 1410   11.   Rs. 30, Rs. 300

12.   Rs. 3, Rs. 0.50   13.   Rs. 300, Rs. 350

14.   Rs. 13, Rs. 17

Exercise − 3.8

1. 18,20   2. 24,32   3. 10,5   4. 4,12
5. 40, 48   5. 23

Exercise − 3.9

1. 90°   2. ∠A = 65°, ∠B = 55°, ∠C = 115°, ∠D = 125°
3. 120°, 30°, 30°   4. 90°, 67 ½ °, 22 ½ °
5. 60°, 30°, 90°

Exercise − 3.10

1. 19 years and 16 years   2. 40 years, 15 years
3. 45 years, 15 years   4. 34 years, 12 years
5. 52 years, 12 years   6. 50 years, 20 years
7. 33 years, 10 years   8. 21 years, 27 years
9. 84, years, 59 years   10. 36 years
11. 42 years, 12 years   12. 45 years
13. 36 years, 6 years   14. 30 years, 6 years
15. 49 year, 7 years   16. 40 years, 10 years
17. 33 years, 9 years   18. 50 years, 20 years

Exercise − 3.11

1. 36   2. 36   3. 93
4. 53   5. 47 or 74   6. 41
7. 95 or 59   8. 54   9. 53

Exercise − 3.12

1. x / y = 3 / 10   2. 7 / 9   3. 3 / 5
4. 3 / 5   5. 12 / 25   6. 5 / 7
7. 5 / 7   8. 15 / 26   9. 35 and 71

Exercise − 3.13

1. Rs. 213   2. Rs. 180   3. Fixed charge = Rs. 250 and rate per Km. = Rs. 4
4. Fixed charge = Rs. 3 and rate per Km. = Rs. 3.5
5. Fixed charge = Rs. 400 and cost per day. = Rs. 30
6. Rs. 160
7. Rent of the house = Rs. 1500 and charge per head per month = 1200

Exercise − 3.14

1. 6 Hour   2. 45 km per Hr   3. 60 km per Hour
4. speed of train = 100 km/h and speed of the car = 120 km/h
5. speed of the car A = 35 km/h and speed of the car B = 25 km/h

Exercise − 3.15

1. l = 40, b = 30   2. 225 days   3. 4 days   4. 12 days

Exercise − 3.16

1. 2 km/h   2. 5 km/h   3. 5 km/h
4. speed of the sailor = 10 km/h and speed of the current = 2 km/h
5. Speed of the boat = 6 km/h and speed of the stream = 2 km/h.